Spring 2021 ECON200C: Discussion 2 - Adverse Selection

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Adverse Selection and Information Asymmetry

- Consider in a market with information asymmetry, there could be potential problems to the participants, e.g. **Lemon Market** (Akerlof 1970)
- For buyers and sellers who have different information, one side with more information could choose the action that benefits them the most, at the expense of the other party
- In response to such potential disadvantages, the side without information might distort their decisions, e.g. quitting the market, and diminishing the market efficiency
- Such problem is so called **adverse selection** problem

Screening and Signaling (Spence 1973)

- To cope with the adverse selection problem, **screening** is one of the strategy
- The idea is that, for the less informed party, it is beneficial to "screen" out some undesired types of the opponents/agents by providing offers that are only accepted by the desired type of the opponents/agents

- A "similar" strategy, **signaling**, also tempts to resolve the information asymmetry
- While in signaling game, the more informed party will send a "signal" (usually costly) to the other side, and the signals will distinguish themselves from the other types

First Best

- An example of the screening problem, we have two types of buyers, $i \in \{H, L\}$, with β chance to be L and $1 - \beta$ chance to be H
- we further assume each i has utility form $\theta_i V(q_i) - T_i$, and $\theta_H > \theta_L$
- seller is going to design the menu of pricing for each buyer, $\{T_i, q_i\}$ to maximize her (expected) revenue
- **First Best**: no information asymmetry - types of buyers are known
- seller's problem

$$\max_{\{T_i, q_i\}} T_i - cq_i$$

$$s.t. \quad \theta_i V(q_i) - T_i >= 0 \text{ (PC)}$$

- Here the PC must be binding since seller could always increases T_i to collect more revenue
- Replace T_i by its binding constraint and solved for q_i by FOC

Second Best

- **Second Best**: information asymmetry - types of buyers are unknown to the seller
- both types could come to seller and seller need to propose a menu of (T_i, q_i) for buyers to choose
- seller's problem becomes:

$$\max_{\{T_i, q_i\}} \beta(T_L - cq_L) + (1-\beta)(T_H - cq_H)$$

$$\begin{aligned} s.t. \quad & \theta_H V(q_H) - T_H >= \theta_H V(q_L) - T_L && \text{(ICH)} \\ & \theta_L V(q_L) - T_L >= \theta_L V(q_H) - T_H && \text{(ICL)} \\ & \theta_H V(q_H) - T_H >= 0 && \text{(PCH)} \\ & \theta_L V(q_L) - T_L >= 0 && \text{(PCL)} \end{aligned}$$

Second Best

- First, check whether FB solution satisfies the constraints
- The answer is no. For high type, ICH is not satisfied

$$\theta_H V(q_L) - T_L > \theta_L V(q_L) - T_L = 0 = \theta_H V(q_H) - T_H$$

So high type will deviate and mimic low type for higher utility

- Second, we consider a **separating contract**, that is to provide different menus to different types of buyers
- (1) Observe that PCH will not bind given PCL and ICH are satisfied:

$$\underbrace{\theta_H V(q_H) - T_H >= \theta_H V(q_L) - T_L}_{ICH} > \underbrace{\theta_L V(q_L) - T_L >= 0}_{PCL}$$

$$\Rightarrow \theta_H V(q_H) - T_H > 0 \text{ (PCH with strict inequality)}$$

Second Best

- (2) By single crossing condition, we should not have two ICs binding at the same time
- single crossing condition

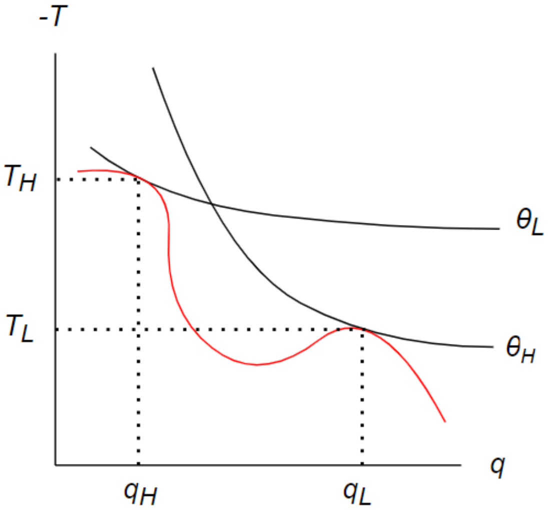
$$\frac{\partial}{\partial\theta}[-\frac{\frac{\partial u}{\partial q}}{\frac{\partial u}{\partial T}}] > 0$$

- This condition is saying, as θ increases, the MRS of the utility should strictly increase. In other words, when we compare low type with high type, high type's MRS should be larger than low type's MRS everywhere.
- each binding IC pins down two points on the same isoutility curve

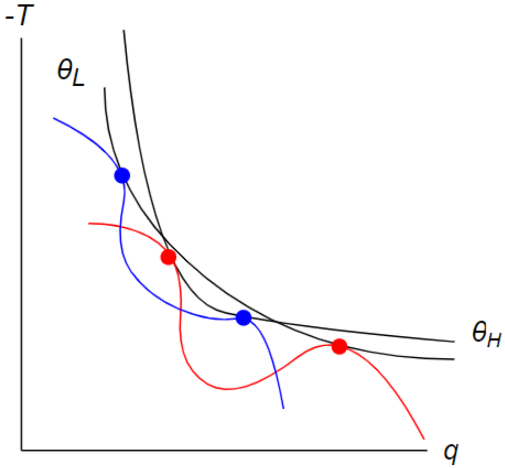
$$\theta_H V(q_H) - T_H = \theta_H V(q_L) - T_L \qquad \text{(ICH)}$$
$$\theta_L V(q_L) - T_L = \theta_L V(q_H) - T_H \qquad \text{(ICL)}$$

- if both ICs are satisfied, we could find two mutual points on two isoutility curves, which violates the single crossing condition

Single Crossing Condition



Single Crossing Condition



Second Best

- So that's why we want to omit one IC and solve the relaxed problem, after we have the solution, we come back and check if the omitted IC is still satisfied
- Since in the FB solution, we have high type whose IC is not satisfied in SB, while low type's IC is not binding, we choose to omit ICL (you need to check this condition for each different problem, as not always "low" type will have slack IC)
- (3) Now, we have PCH is slack, and omit ICL. For the remaining IC, we argue that ICH should bind, otherwise seller could increase T_H to collect revenue; similarly, PCL should bind, otherwise increase T_L
- Then, we could replace T_L and T_H by their binding constraints into the optimization problem and solve for $\{q_H, q_L\}$

Second Best

- (4) For the proposed solution $\{q_i, T_i\}$, we need to plugin ICL to check if it is satisfied (Important!!!)
- If yes, we are done. If not, we omit ICH and bind ICL to solve for any solution.(This is not going to be optimal in this example, as seller could always increasing T_L a log which relaxes ICL but does not violate ICH, and it will strictly increase the revenue, which makes the solution suboptimal. In other setup, it could be possible to bind any type's IC to find the optimal contract) Also, we need to check for the case where ICH and ICL are both slack.(Kuhn-Tucker or Lagrange multiplier method could solve for this optimization problem generally)
- Third, we should also consider different potential contracts. e.g. A **pooling contract** that satisfies both PCs for high and low types; a **"screening" contract** that only satisfies one type's PC but not another one, so that only one type will participate the deal.

Question for Practice

We have agent's cost type is θ then his net utility gain from providing h hours of service and receiving wage w would be $u(w, h) = w - \theta h$. The principal's net benefit from paying w for h hours of service from either type would be $6\sqrt{h} - w$. Agent's outside option is a net utility equal to 1. Suppose that the agent's cost type $\theta \in \{3, 5\}$, and the principal thinks that the associated probabilities are $P(\theta = 3) = \frac{3}{4}$ and $P(\theta = 5) = \frac{1}{4}$. Find the second best contract that maximizes the principal's expected net benefit from her business with the agent. Hours of service are observable and verifiable.

Numerical Example

- Again, first consider FB solution. In FB, we want to bind both PCs

$$w_L - \theta_L h_L = 1$$
$$w_H - \theta_H h_H = 1$$

Easy to solve for maximization problem $\max_{h_i} = 6\sqrt{h_i} - (1 + \theta_i h_i)$
Solutions are:

$$h_L = 1 \qquad w_L = 4$$
$$h_H = \frac{9}{25} \qquad w_H = \frac{14}{5}$$

- check if ICs are satisfied in SB problem:

$$w_L - \theta_L h_L >= w_H - \theta_L h_H$$
$$w_H - \theta_H h_H >= w_L - \theta_H h_L$$

- We can find ICL is not satisfied, low type wants to deviate

Second Best: Separating Contract

- Now, as we already argued, PCL is not binding given ICL and PCH are satisfied (check math yourself), and we could omit ICH for now; so we set PCH and ICL binding
- we rewrite the maximization problem with PCH and ICL replacing the w_i: $\max_{\{h_i\}} \frac{3}{4}(6\sqrt{h_L} - w_L) + \frac{1}{4}(6\sqrt{h_H} - w_H)$ where $w_H = 1 + \theta_H h_H, w_L = (1 + \theta_H h_H) + \theta_L(h_L - h_H)$
- the solution yields $h_L = 1,\ h_H = 9/121,\ w_L = \frac{502}{121},\ w_H = \frac{166}{121}$
- Check that ICH is satisfied for the solution, so it is a valid contract
- Calculate principal's expected payoff $\mathbb{E}\pi^S = \frac{3}{4}(6 - 502/121) + \frac{1}{4}(6 * \frac{3}{11} - 161/121) = 1.455$

Second Best: Pooling Contract

Only need to provide one contract that satisfies both PCs

$$max_{w_p,h_p} 6\sqrt{h_p} - w_p$$
$$s.t. w_p - 3h_p >= 1 \text{ (slack)}$$
$$w_p - 5h_p >= 1 \text{ (binding)}$$

Solve for $h_p = \frac{9}{25}, w_p = \frac{14}{5}$
Calculate principal's expected payoff $\mathbb{E}\pi^P = 6 * \frac{3}{5} - \frac{14}{5} = \frac{4}{5}$

Second Best: Screening Contract

- Consider only making an feasible offer to the more efficient agent, in this case, low type
- So only need to satisfy PCL: $w_{sc} - 3h_{sc} = 1$ and solve for maximization problem
- Solution is $h_{sc} = 1,\ w_{sc} = 4$
  principal's expected payoff $\mathbb{E}\pi^{SC} = \frac{3}{4}(6 - 4) = 1.5$
- Since it yields the highest payoff, so this is the SB optimal contract.