Spring 2021 ECON200C: Discussion 3 - Insurance Market (R&S (1976))

April 16, 2021

Setup

- We have two kinds of consumer, high type has probability for accident p_H, and low type has that for p_L, with $p_H > p_L$
- Again, the proportion of high type is η, that of low type is $1 - \eta$
- Consumer pays premium α_1, and gets reimbursement $\hat{\alpha}_2$, we define $\alpha_2 = \hat{\alpha}_2 - \alpha_1$ the net payment to consumer
- We have a monopoly insurance company, who is risk neutral, and her utility is the revenue
- Thus, insurer maximize expected revenue:

$$\max_{\alpha_1^H, \alpha_2^H, \alpha_1^L, \alpha_2^L} \eta[\alpha_1^H - p_H \alpha_2^H] + (1 - \eta)[\alpha_1^L - p_L \alpha_2^L]$$

First Best

- Again, we first consider FB case
- In FB, insurer could provide each type of consumer with different contract, as information symmetry is assumed
- We only consider PCs for each type:

$$\begin{aligned}(1-p_H)U(W-\alpha_1^H)+p_H U(W-d+\alpha_2^H)\\ \geq (1-p_H)U(W)+p_H U(W-d) \qquad PC_H\end{aligned}$$

$$\begin{aligned}(1-p_L)U(W-\alpha_1^L)+p_L U(W-d+\alpha_2^L)\\ \geq (1-p_L)U(W)+p_L U(W-d) \qquad PC_L\end{aligned}$$

- We could argue that both PCs need to be binding

Second Best

- For SB, we need to consider all 4 constraints:

$$(1-p_H)U(W-\alpha_1^H)+p_H U(W-d+\alpha_2^H) \geq (1-p_H)U(W)+p_H U(W-d) \qquad PC_H$$

$$(1-p_L)U(W-\alpha_1^L)+p_L U(W-d+\alpha_2^L) \geq (1-p_L)U(W)+p_L U(W-d) \qquad PC_L$$

$$(1-p_H)U(W-\alpha_1^H)+p_H U(W-d+\alpha_2^H) \geq (1-p_H)U(W-\alpha_1^L)+p_H U(W-d+\alpha_2^L) \qquad IC_H$$

$$(1-p_L)U(W-\alpha_1^L)+p_L U(W-d+\alpha_2^L) \geq (1-p_L)U(W-\alpha_1^H)+p_L U(W-d+\alpha_2^H) \qquad IC_L$$

Second Best

- **Claim 1**: In any separating equilibrium, $\alpha_1^L < \alpha_1^H$, $\alpha_2^L < \alpha_2^H$, or equivalently, $W_1^L > W_1^H$, $W_2^L < W_2^H$
- Proof by contradiction: First, show that we cannot have $W_1^i < W_1^j$, $W_2^i < W_2^j$, otherwise, type i will deviate
- Then assume $W_1^L < W_1^H$, $W_2^L > W_2^H$, you need to combine two ICs and show it is not possible

- **Claim 2**: In any separating equilibrium, exactly one IC is binding
- Assume both ICs are slack, then both PCs should bind. Show that in this case, there will be $\alpha_1^L < 0$ which is not allowed
- Then assume both ICs are binding, not possible as shown in Claim 1 proof.

Second Best

- **Claim 3**: For any optimal separating contract, exactly one PC is binding.
- Assume both PCs are slack, then since by Claim 2, one IC is binding, assuming for type i, then you can increase profit by binding PC for type $j \neq i$
- Then assume both PCs are binding, we have shown in this case $\alpha_1^L < 0$

- **Claim 4**: There is no separating contract under which PC_H is binding
- Assume PC_H is binding, you can find again $\alpha_1^L < 0$

- **Claim 5**: For any optimal separating contract, IC_H is binding
- Assume IC_H slack, then IC_L must bind, then from binding IC_L and PC_L(Claim 4), we can still increase profit

Optimal Separating Contract

$$\max_{\alpha_1^H, \alpha_2^H, \alpha_1^L, \alpha_2^L} \eta[\alpha_1^H - p_H \alpha_2^H] + (1-\eta)[\alpha_1^L - p_L \alpha_2^L]$$

$$\text{s.t. } PC_L, IC_H \text{ are binding}$$

- You should find in optimal separating contract, insurer will provide full insurance to the High Risk Type and incomplete insurance to the Low Risk Type
- From intuition, this is because High Risk Type has the incentive to mimic Low Risk Type, so insurer has to provide a full insurance to make High Risk Type not deviate
- Also consider pooling contract, and screening contract and argue whether they could be a optimal contract (graphs are OK)