Spring 2021 ECON200C: Discussion 4 - Moral Hazard

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Moral Hazard Versus Adverse Selection

- Consider our insurance example, with two types, θ_H and θ_L
- The information asymmetry is about the types of consumers, in other words, the *type* is not contractible/verifiable
- The contract design problem under this case is called **Adverse Selection**

- Now instead of type, introduce consumer's effort level, *careless* and *careful*
- The information asymmetry is about the effort level of the consumer, in other words, the *effort* is not contractible/verifiable (Here we need to assume outcome is not perfectly correlated/not a perfect signal of effort level)
- The contract design problem under this case is called **Moral Hazard**

Environment

- A principal needs to hire an agent to work by signing a contract
- output level $q(e)$ depends on the effort level, e, chosen by the agent
- agent's utility $u(w, e)$ is typically increasing in wage, w, decreasing in effort, e, as effort is costly. Outside option has reservation value $\bar{u} = 0$
- Principal has payoff $v(q - w)$ and outside option also 0

- **Timeline**: The principal offers a contract in the form of a schedule of wages (e.g. if certain condition is satisfied, then pay certain level of wage)
- agent decides to take or reject the offer, and if accepts, agent decides the effort level and payoffs are realized; if rejects, both parties receive their outside options

First Best

- In FB, full information is assumed so principal could observe agent's effort level
- As $q(e)$ only depends on e, principal will set the schedule of wages based on effort, $w(e)$
- Principal's problem:

$$max_{w(e)} v(q(e) - w(e))$$
$$\text{s.t. } u(w(e), e) \geq \bar{u} \tag{PC}$$

- Essentially, it is same to find the optimal effort level e^{FB} to induce the agent to implement:

$$e^{FB} = \arg\max_e v(q(e) - \tilde{w}(e))$$

where $\tilde{w}(e)$ satisfies:

$$u(\tilde{w}(e), e^{FB}) = \bar{u}$$

FB Solution

- Take FOC respective to e for the objective function, it yields:

$$v'(q(e) - \tilde{w}(e))(\frac{\partial q}{\partial e} - \frac{\partial \tilde{w}(e)}{\partial e}) = 0$$

- With normal assumption that $v'(\cdot) > 0$, we have

$$\frac{\partial q}{\partial e} = \frac{\partial \tilde{w}(e)}{\partial e}$$

- Since $\tilde{w}(e)$ satisfies binding PC, it should also satisfy the first order condition of PC (taking full derivative respective to e):

$$\frac{\partial u}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial e} + \frac{\partial u}{\partial e} = 0$$
$$\frac{\partial \tilde{w}}{\partial e} = -(\frac{\partial u / \partial e}{\partial u / \partial \tilde{w}})$$

FB solution

- Combine the FOC and PC conditions, we have:

$$\underbrace{\left(\frac{\partial u}{\partial \tilde{w}}\right)}_{\substack{\text{marginal utility}\\ \text{of wage}}} \quad \underbrace{\left(\frac{\partial q}{\partial e}\right)}_{\substack{\text{marginal output}\\ \text{in effort}}} \quad = \quad \underbrace{-\left(\frac{\partial u}{\partial e}\right)}_{\substack{\text{marginal disutility}\\ \text{of the effort}}}$$

- Interpretation: at the optimal effort level e^{FB}, the marginal utility of effort to the agent that results if he kept all the additional output to himself, equals the marginal disutility of the effort.
- Summary: For each effort level, calculate what's the lowest wage level to induce the effort level, and also what will be the expected production and profit under this effort level. Pick the effort level that yields highest profit and the corresponding effort level and wage.

Example (2019 Midterm)

Consider a company hires a salesman to sell product. If a salesman exerts high effort, he will set with probability 0.8. If he exerts low effort, he will succeed with probability 0.4. The company will make a profit of 2000 if the sale is made. The cost of low effort is 40 and high effort is 85. The agent is risk neutral and his utility is the payment from company subtracted by the effort cost.

Q: What is first best effort level and payment if the company has all the bargaining power?

Example (2019 Midterm)

To solve for FB, your job is to find what is the optimal effort level. It is easy to do this if you have a finite effort space, just check the profit one by one.

- High Effort: Let PC binding, that is $w(e_H) - 85 = 0$, so $w(e_H) = 85$
- $\mathbb{E}\pi^H = 2000 * 0.8 + 0 * 0.2 - w(e_H)$, solve for $\mathbb{E}\pi^H = 1515$

- Low Effort: Let PC binding, that is $w(e_L) - 40 = 0$, so $w(e_L) = 40$
- $\mathbb{E}\pi^L = 2000 * 0.4 + 0 * 0.6 - w(e_L)$, solve for $\mathbb{E}\pi^H = 760$

- So high effort is optimal, and FB contract should enforce high effort with wage equals to 85 for high effort.

Second Best

- In SB, there exists asymmetric information on the effort level
- Since output is a probabilistic function of effort, output level does not fully reveal the effort level. So a contract could not depend on effort anymore.
- SB contract could only depend on the realized output level, $\tilde{w}(q)$
- Problem becomes:

$$
\begin{aligned}
&\max_{\tilde{w}(q)} \mathbb{E}[V(q(\tilde{e}) - \tilde{w}(q(\tilde{e}))] \\
&\text{s.t. } \mathbb{E}[u(\tilde{e}, \tilde{w}(q(\tilde{e}))] \geq \bar{u} && \text{(PC)} \\
&\tilde{e} = \arg\max_{e} \mathbb{E}[u(e, \tilde{w}(q(e))] && \text{(IC)}
\end{aligned}
$$

SB Solution

- Grossman and Hart (1983) proposes the solution for the problem in finitely dimensional effort space E and outcome space Q
- **Step 1** For any given $e \in E$, solve for wage schedule that minimizes the expected payment to the agent and induces that effort level:

$$\min_{\tilde{w}(q)} \mathbb{E}[\tilde{w}(q(e))]$$

$$\text{s.t. } \mathbb{E}[u(e, \tilde{w}(q(e))] \geq \bar{u} \qquad \text{(PC)}$$

$$u(e, \tilde{w}(q(e)) \geq u(e', \tilde{w}(q(e')) \text{ for any } e' \neq e \qquad \text{(IC)}$$

- **Step 2** Choose the effort level that maximizes the payoff, given the wage schedules drawn from the whole set of wage functions $\tilde{w}(E)$ derived in step 1

$$\max_{\tilde{w} \in \tilde{w}(E)} \mathbb{E}[V(q(\tilde{e}) - \tilde{w}(q(\tilde{e}))]$$

Risk Neutral Agent

- we could simplify the problem to the linear case
- principal utility $v(q, w) = q - w$ and agent utility $u(e, w) = w - K(e)$ where $K(e)$ denotes the cost function of effort with $K'(\cdot) > 0$
- $\mathbb{E}V = \mathbb{E}(q - w)$ and $\mathbb{E}u = \mathbb{E}[w] - K(e)$
- Combine both utility functions, we have $\mathbb{E}V = \mathbb{E}[q(e)] - K(e) - \mathbb{E}u$
- Principal needs to maximize this $\mathbb{E}V$, that is fixing the level of agent's utility to his reservation level $\bar{u}$, and pick the effort level that yields the highest profit
- This is same as our FB case, but how to design the contract?

Risk Neutral Agent

- One solution is "sell-the-store", charging a lump-sum value t and leave all the leftover output to the agent
- Essentially, this transfer all the risk to the agent, thus agent could maximize the output with consideration of all risk, instead of partial risk which would distort the decision (similar to linear tax over income which is distortionary but lump-sum tax is not)
- **sell-the-store**: $w(q) = q - t$, so principal tries to maximize the payment t and agent needs to maximize $\mathbb{E}u = \mathbb{E}(q(e)) - K(e) - t$
- It is easy to see, agent will always pick the optimal effort that yields highest output, then anticipating this, principal could set t equal to the expected surplus from optimal effort

Risk Averse Agent

- If agent is risk averse, then the payment from principal to agent need to compensate not only the cost of effort, but also the risk from varying wage
- "sell-the-store" won't work since agent will not choose the effort level that generates highest expected payoff, but choose the effort level that also yields lower risk of the outcome (e.g. low expected payoff but certain level of output)
- How to solve? Apply Grossman and Hart (1983) method. There are two possible types of solutions: (1) flat wage that provides same wage regardless outcome and induces the effort level that needs lowest cost; (2) wage schedule that provides large enough wedge between good and bad outcomes to incentivize higher effort level(not always highest level) that yields the maximized profit

Example Continued

Consider a company hires a salesman to sell product. If a salesman exerts high effort, he will set with probability 0.8. If he exerts low effort, he will succeed with probability 0.4. The company will make a profit of 2000 if the sale is made. The cost of low effort is 40 and high effort is 85. The agent is risk neutral and his utility is the payment from company subtracted by the effort cost.

Q: What is optimal contract given only outcome is observable?
A: Sell the store with fixed fee equal to full surplus in FB. Agent will choose high effort to maximize the output.

Example Continued - Risk Averse Agent

Consider a company hires a salesman to sell product. If a salesman exerts high effort, he will set with probability 0.8. If he exerts low effort, he will succeed with probability 0.4. The company will make a profit of 2000 if the sale is made. The cost of low effort is 40 and high effort is 85. The agent is now risk averse and his utility is $\sqrt{w} - K(e)$, where w is the wage and $K(e)$ is the cost of effort.

Q: What is the SB optimal contract?

Example Continued - Risk Averse Agent

Principal's problem: Choose between a wage schedule that induces high effort or a flat wage that induces low effort

High Effort:

$$\max_{w_S, w_F} \mathbb{E}[q(e_H) - \tilde{w}(q)] = (2000 - w_S) * 0.8 + (0 - w_F) * 0.2$$

$$\text{s.t. } 0.8\sqrt{w_S} + 0.2\sqrt{w_F} - 85 \geq 0 \tag{PCH}$$

$$0.8\sqrt{w_S} + 0.2\sqrt{w_F} - 85 \geq 0.4\sqrt{w_S} + 0.6\sqrt{w_F} - 40 \tag{ICH}$$

Low Effort:

$$\max_{w} \mathbb{E}[q(e_L) - \tilde{w}(q)] = 2000 * 0.4 + 0 * 0.6 - w = 800 - w$$

$$\text{s.t. } \sqrt{w} - 40 \geq 0 \tag{PCL}$$

Example Continued - Risk Averse Agent

Note: For Low Effort case, the reason we could set a flat wage is because this will ensure the lowest wage ($w_S > w_L$ will provide unnecessary incentive for high effort; $w_S < w_L$ will be punished for imposing extra risk on the agent), and we don't need ICL since it will be always slack as high effort is costly

For Low Effort, it is straightforward to solve for $w = 1600$ from binding PCL and principal expected payoff equals -800

Example Continued - Risk Averse Agent

For High Effort, we could start from binding both PCH and ICH (ICH is requiring for a wedge between w_S and w_F large enough to incentivize high effort, and PCH is requiring for a weighted average of w_S and w_F to be higher than reservation value; to minimize overall wage expenditure, it is same to set the two objects as low as possible)

Binding PCH and ICH you will get $\sqrt{w_S} = 107.5$, $\sqrt{w_F} = -5 < 0$, not valid; that means at most we set $w_F = 0$, and we need to try whether we could bind PCH or ICH by omitting the other one first and then check if it is still satisfied

Example Continued - Risk Averse Agent

First, we let PCH binding and omit ICH, solve for $\sqrt{w_S} = 106.25$, then plug in ICH and we find ICH is not satisfied

Then, we try ICH binding and omit PCH, solve for $\sqrt{w_S} = 112.5$, and plug in PCH and it is still satisfied, so this is the contract for High Effort. The principal's payoff from wage schedule $w_F = 0, \sqrt{w_S} = 112.5$ is -8525.

Compare the High Effort and Low Effort, we found Low Effort is optimal. So the SB contract is to offer a flat wage of 1600 and induce low effort.