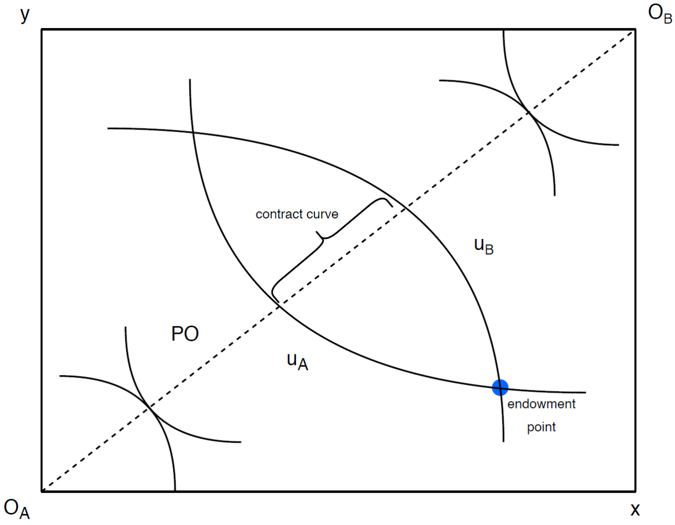
Spring 2021 ECON200C: Discussion 5 - General Equilibrium

May 21, 2021

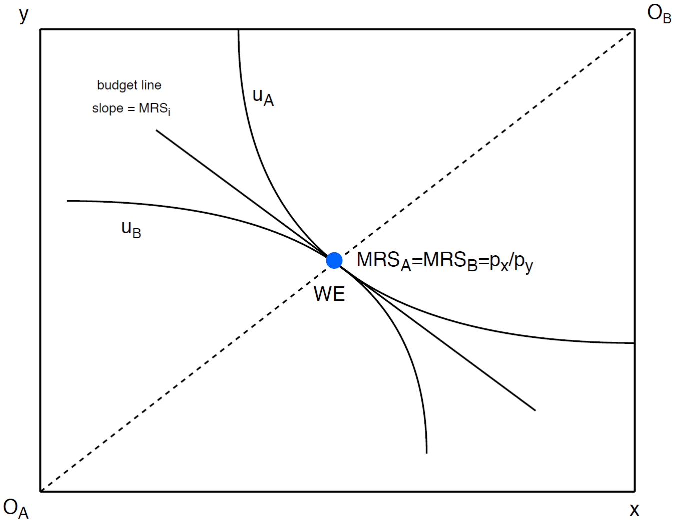
Edgeworth Box

- two goods x, y
- two agent A, B
- endowment (ω_x^A, ω_y^A), (ω_x^B, ω_y^B)
- price p_x(normalize to 1), p_y

Edgeworth Box



Edgeworth Box



Pareto Optimum

- when the utility function is convex and preferences are locally non-satiated, we can directly look at the set characterized by:

$$MRS^A_{x,y} = MRS^B_{x,y} \longrightarrow \frac{MU^A_x}{MU^A_y} = \frac{MU^B_x}{MU^B_y}$$

- A more general to find the PO set is to maximize one agent's utility while fixing another agent's utility:

$$\max_{x_A, y_A} u_A(x_A, y_A)$$
$$\text{s.t. } u_B(x_B, y_B) = \bar{u}$$
$$\sum_i x_i = \sum_i \omega^i_x$$
$$\sum_i y_i = \sum_i \omega^i_y$$

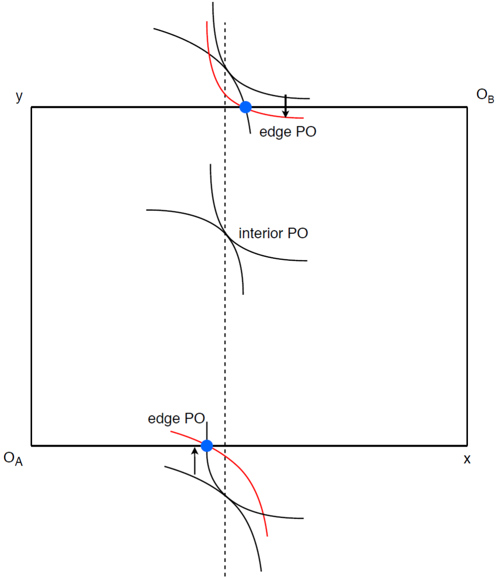
Pareto Optimum

- You could solve the problem by setting up Lagrangian and FOCs:

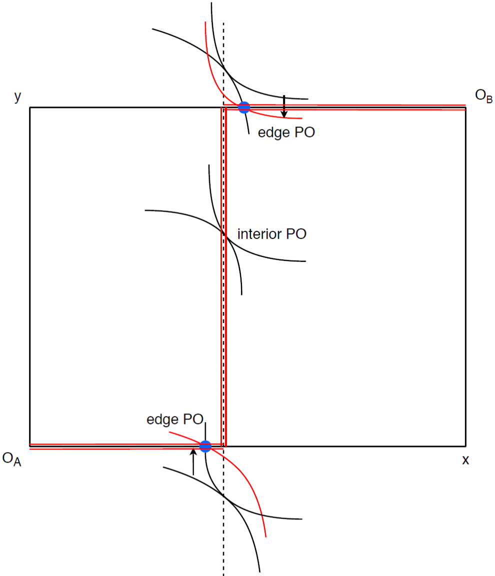
$$\mathcal{L} = u_A(x_A, y_A) + \lambda[u_B\Big(\sum_i \omega_x^i - x_A, \sum_i \omega_y^i - y_A\Big) - \bar{u}]$$

- Essentially, when assumptions are satisfied, your FOCs are same as $MRS^A_{x,y} = MRS^B_{x,y}$
- The solution is a set of allocations $\{x_i, y_i\}_{PO}$
- However, this only solves for interior PO; we should always check for potential edge PO by looking at MRS

Pareto Optimum



Pareto Optimum



PO but not WE

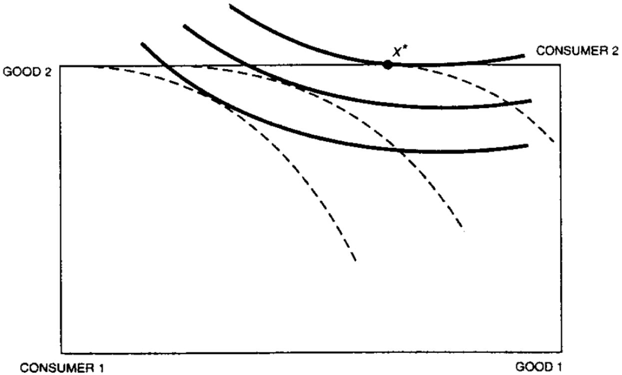


Figure 17.6 **Arrow's exceptional case.** The allocation $\mathbf{x}^*$ is Pareto efficient but there are no prices at which x^* is a Walrasian equilibrium.

Contract Curve

- Recall that contract curve characterize the subset of PO that is feasible to the agents.
- Find allocations in PO that provide higher utility for both agents than those from the endowment point.

$$\{x_i, y_i\}_{CC} = \{(x_i, y_i) \in PO | u_i(x_i, y_i) \geq u_i(\omega_x^i, \omega_y^i) \ \forall i\}$$

Walrasian Equilibrium

$$\max_{x_i, y_i} u_i(x_i, y_i)$$
$$\text{s.t. } p_x x_i + p_y y_i \leq p_x w_x^i + p_y w_y^i$$

- This solves for the demand functions of each agent $x_i(\mathbf{p}, \boldsymbol{\omega})$, $y_i(\mathbf{p}, \boldsymbol{\omega})$
- Then, we could either use excess demand function or market clear condition to find the supporting price vector

$$\sum \mathbf{z}(x_i, y_i) = 0 \quad \text{or} \quad \begin{array}{c} \sum_i x_i = \sum_i \omega_x^i \\ \sum_i y_i = \sum_i \omega_y^i \end{array}$$

Example: Cobb-Douglas

- Consider the two consumer, two product exchange economy. We assume consumers have Cobb-Douglas utility with $\alpha = \frac{1}{2}$, that is $u_i(x_i, y_i) = x_i^{1/2} y_i^{1/2}$ for $i \in \{A, B\}$. The endowment vector is $(\omega_x^A, \omega_y^A, \omega_x^B, \omega_y^B) = (2, 0, 0, 2)$. Denote the price of x as 1, and the price of y as p.

- Q: What are the quantities demanded at price p? Verify that Walras' Law holds.

Example: Cobb-Douglas

Solve for the maximization problem:

$$\max_{x_i, y_i} x_i^{1/2} y_i^{1/2}$$

$$\text{s.t. } p_x x_i + p_y y_i \leq p_x w_x^i + p_y w_y^i$$

The FOC for consumer A is

$$\frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}} = \lambda p_x \tag{1}$$

$$\frac{1}{2} x^{\frac{1}{2}} y^{-\frac{1}{2}} = \lambda p_y \tag{2}$$

$$p_x(2 - x) = p_y y \tag{3}$$

Example: Cobb-Douglas

(1) and (2) gives us

$$p_x x = p_y y$$

Recall this is the property of Cobb-Douglas utility: α gives the ratio of the total expenditure spent on each good, and here $\alpha = 1/2$

Since we assume $p_x = 1$, combine with (3) we have $x_A = 1$, $y_A = \frac{1}{p_y}$
Check that A's total expenditure equal to her budget $= 2$.

Similarly, solve for B's demand functions: $x_B = p_y$, $y_B = 1$

Walras' Law holds since we could check the sum of the values of the excess demand for both goods:

$$p_x[(x_A - 2) + (x_B - 0)] + p_y[(y_A - 0) + (y_B - 2)] = 0$$

Example: Cobb-Douglas

We could move on to solve for the Walrasian Equilibrium prices and quantities by clearing the markets:

$$x_A + x_B = \omega_x^A + \omega_x^B$$
$$y_A + y_B = \omega_y^A + \omega_y^B$$

This solves for

$$p_y = 1, x_A = 1, \ y_A = 1$$
$$x_B = 1, \ y_B = 1$$

Example: Leontief

- Now consider the consumers' utility function has the form:

$$u_i(x_i, y_i) = \min(ax_i, by_i)$$

- What will be the demand functions and WE?

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- Recall that with Leontief utilities, the consumer will choose consumption bundle in the form of $ax_i = by_i$, so the demand functions are:

$$x_A = \frac{2a}{a + bp_y},\ y_A = \frac{2b}{a + bp_y},\ x_B = \frac{2ap_y}{a + bp_y},\ y_B = \frac{2bp_y}{a + bp_y}$$

  and WE price is $p_y = \frac{1-a}{b-1}$, note this is not defined in many cases, why is this case?

Example: Linear

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- Now consider the consumers' utility function has the form:

$$u_i(x_i, y_i) = ax_i + by_i$$

- What will be the demand functions and WE?

- With linear utilities, the consumers will choose the cheaper good per utility unit to maximize their utilities, i.e, they only buy goods x if $\frac{MU_x}{p_x} = a > \frac{b}{p_y} = \frac{MU_y}{p_y}$ and vice versa.

  WE exists for $p_y = \frac{b}{a}$ and any allocation such that $\sum x_i = \sum y_i = 2$ is an WE allocation.

Example: Non-Walrasian Equilibrium

- Go back to the Cobb-Douglas example, what will happen if the government imposes a price ceiling of $p = \frac{1}{2}$? Is there any WE?

Example: Non-Walrasian Equilibrium

- Go back to the Cobb-Douglas example, what will happen if the government imposes a price ceiling of $p = \frac{1}{2}$? Is there any WE?

- There will be an excess supply of x and an excess demand for y.

  With the price ceiling, a WE does not exist. But we could have other forms of equilibria if there are methods to deal with the excess supply and excess demand.

  E.g., exporting the excess supply abroad and ration the excess demanded goods by performing on a first-come, first-served basis.

Example: Non-Walrasian Equilibrium