Spring 2021 ECON200C: Discussion 6 - Mechanism Design

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Introduction

- The design of the institutions could have profound impact on the way agents behave, e.g. first-price auction vs. second-price auction
- mechanism design aims to design the institution satisfying certain objectives, assuming that individuals 1) interact strategically, and 2) hold private information
- each individual has a message (or strategy) space
- designer chooses decision rule as a function of the messages received
- transfers among the individuals are allowed

A General Setting

- individuals denoted by $i \in N = \{1, 2, ..., n\}$ in a finite group
- Society decision space is denoted D, and decisions are represented as d, d'.
- private information represented by a type $\theta_i \in \Theta_i$. We have a type vector $\theta = (\theta_1, ..., \theta_n)$. And let $\Theta = \Theta_1 \times ... \times \Theta_n$
- individual preference is a private value dependent only on type and her decision, $v_i : D \times \Theta_i \longrightarrow \mathbb{R}$.

Example

Public Project Example:

A society decision on building a public project or not at a cost c. The cost needs to be equally divided. Here $D = \{0, 1\}$ with 0 representing no building and 1 representing building.
Individual i's value from the project is her type, θ_i.
Thus, the utility function could be represented by:

$$v_i(d, \theta_i) = d\theta_i - d\frac{c}{n}$$

Decision Rules and Efficient Decisions

- A *decision rule* is a mapping $d : \Theta \longrightarrow D$, indicating a choice $d(\theta) \in D$ as a function of θ.
- A decision rule $d(\cdot)$ is *efficient* if

$$\sum_i v_i(d(\theta), \theta_i) \geq \sum_i v_i(d'(\theta), \theta_i)$$

  for all θ and $d' \in D$.
- In the public project example, the efficient decision rule is $d(\theta) = 1$ when $\sum_i \theta_i > c$ and $d(\theta) = 0$ when $\sum_i \theta_i < c$.

Transfer functions and Social Choice Functions

- transfers provide necessary incentives to allocate efficiently, e.g. tax or subsidize individuals
- a transfer function $t : \Theta \longrightarrow \mathbb{R}^n$ represents the payment that i receives(pays) based on the **announcement** of types θ.
- A pair d, t will be referred to as a *social choice function*, and denote it by $f(\theta) = (d(\theta), t(\theta))$.
- The *quasi-linear* utility that i receives in the end is

$$u_i(\hat{\theta}, \theta_i, d, t) = v_i(d(\hat{\theta}), \theta_i) + t_i(\hat{\theta})$$

where $\hat{\theta}$ is the announced vector of types and i's true type is θ_i.

Transfer Functions: Feasibility and Balance

- A transfer function is *feasible* if $0 \geq \sum_i t_i(\theta)$ for all θ.
- If not feasible, then the society needs some outside source of transfers.
- A transfer function is *balanced* if $\sum_i t_i(\theta) = 0$ for all θ.
- If only feasible but not balanced, then there is some net loss in utility to society relative to an efficient decision with no transfers at some θ.

Mechanisms

- A *mechanism* is a pair M, g, where $M = M_1 \times ... \times M_n$ is a cross product of message or strategy spaces and $g : M \longrightarrow D \times \mathbb{R}^n$ is an outcome function.
- For each profile of messages $m = (m_1, ..., m_n)$, the resulting decision and transfers are represented by

$$g(m) = (\underbrace{g_d(m)}_{\text{decision}}, \underbrace{g_{t,1}(m), ..., g_{t,n}(m)}_{\text{transfers}})$$

- Notice there is no type specific design, since mechanism designs for all θ. Once the preferences of the individuals are specified, then a mechanism induces a game.

Dominant Strategies

- We start by identifying situations where individuals have unambiguously best strategies(messages).
- A strategy $m_i \in M_i$ is a *dominant strategy* at $\theta_i \in \Theta_i$ if

  $$v_i(g_d(m_{-i}, m_i), \theta_i) + g_{t,i}(m_{-i}, m_i) \geq v_i(g_d(m_{-i}, \hat{m}_i), \theta_i) + g_{t,i}(m_{-i}, \hat{m}_i)$$

  for all m_{-i} and .
- A social choice function $f = (d, t)$ is *implemented* in dominant strategies by the mechanism (M, g) if there exist functions $m_i : \Theta_i \longrightarrow M_i$ such that $m_i(\theta_i)$ is a dominant strategy for each i and $\theta_i \in \Theta_i$ and $\underbrace{g(m(\theta)) = f(\theta)}_{\text{implementation}}$ for all $\theta \in \Theta$.

Direct Mechanisms

- A *direct mechanism* is the special case that uses the social choice function $f = (d, t)$ as the mechanism, that is $M_i = \Theta_i$ and $g = f$.
- A direct mechanism is *dominant strategy incentive compatible* if θ_i is a dominant strategy at θ_i for each i and $\theta_i \in \Theta_i$.
- A social choice function is *strategy-proof* if it is dominant strategy incentive compatible.

The Revelation Principle

- **The Revelation Principle for Dominant Strategies:** If a mechanism (M, g) implements a social choice function $f = (d, t)$ in dominant strategies, then the direct mechanism f is dominant strategy incentive compatible.

- Note: the Revelation Principle allows us to restrict our attention to the set of direct mechanisms while finding social choice functions **implemented in dominant strategies**.

Groves' Schemes

- We start with some efficient decision rule d and then asks what form of transfers are necessary so that d, t is dominant strategy incentive compatible.
- The resulting social choice functions are referred as **Groves' schemes**.

Groves' Schemes

Theorem

If d be an efficient decision rule and for each i there exists a function $x_i : \bigtimes_{j \neq i} \Theta_j \longrightarrow \mathbb{R}$ such that

$$t_i(\theta) = x_i(\theta_{-i}) + \sum_{j \neq i} v_j(d(\theta), \theta_j)$$

then (d, t) is dominant strategy incentive compatible.

Proof skipped.

The Pivotal Mechanism

- One simple version of the Groves schemes is the **pivotal mechanism** described by Clarke.
- Let $x_i(\theta_{-i}) = -\max_{d \in D} \sum_{j \neq i} v_j(d, \theta_j)$, then the transfer function becomes

$$t_i(\theta) = \sum_{j \neq i} v_j(d(\theta), \theta_j) - \max_{d \in D} \sum_{j \neq i} v_j(d, \theta_j)$$

The Pivotal Mechanism

- The transfer is always non-positive and thus always feasible.
- If i's presence makes no difference in the maximizing choice of d, then $t_i(\theta) = 0$.
- Otherwise, i is pivotal, then t_i represents the loss in the value imposed on other individuals due to the i's presence.
- **Internalizing Externalities**: each individual's transfer function takes into account the marginal social impact (on other individuals) made by her announcement of θ_i.

Bayesian Mechanism Design

- We only focus on direct mechanism design with dominant strategy incentive compatible cases.
- This is a very strong condition as it requires truthful revelation of preferences being dominant strategy.
- If we introduce probabilistic beliefs over the types of other individuals, the mechanism design problem is called *Bayesian Mechanism Design*.
- It weakens the requirement of dominant strategy incentive compatibility to a Bayesian incentive compatibility condition.

Implementation

- Both Direct and Bayesian Mechanism Design use the revelation principle as a tool.
- It only relates one equilibrium of the mechanism to the truthful strategies.
- Other equilibria could exist rather than direct mechanism.
- The *implementation* literature keeps track of all equilibria and works with the space of indirect mechanisms.