ECON 200C, Spring 2021

Homework 2

Due: 4/23/2021, before the beginning of the discussion session

Problem 1. Consider the Adverse Selection Example in the lecture. Show that if the utility function satisfies the single-crossing property, one of the ICs is not binding. (Hint: Show the single crossing condition with the utility form given in the example, argue that if this holds, then two ICs could not binding at the same time)

Problem 2. Consider a monopoly problem where the monopolist has one unit of an indivisible good for sale, at zero cost. The buyer could have two types, $\{\theta_H, \theta_K\}$ with following utilities: $u_L = \theta_L - T$ and $u_H = \log(\theta_H - T)$, where T is the transfer that the buyer pays to the monopolist seller. The buyer could choose not to buy the good, giving an outside option of 0. We further assume that $\theta_H - 1 > \theta_L$ (this implies that the seller prefers to face the high type buyer).

(a) What will be the First Best outcome?

(b) Show that the seller could implement the FB outcome by using a random pricing scheme.

Problem 3. Consider the Adverse Selection Example in the lecture. Assume now there are three types, $\theta_H > \theta_M > \theta_L$ with probabilities $\beta_H, \beta_M, \beta_L = 1 - \beta_H - \beta_M$. Buyers preferences are presented by $u_i(\theta, q, T) = \theta_i V(q) - T$, where V is a strictly increasing and strictly concave function. Seller profit is given by $\pi = T - cq$.

(a) Find FB contract and seller's profit.

(b) Write down the SB problem for seller, including all ICs and PCs. Argue whether certain constraints are binding. Find the optimal SB contract.

Problem 4. (Rothschild, Stiglitz 1976) For the insurance market with two types of consumers as discussed in class

(a) Solve for the optimal contract(s) offered by a monopolist. (Both FB and SB cases)

(b) What if the insurance market is perfectly competitive? (You only need to set up the maximization problem and list constraints in the SB case)

Problem 5. (Optional with extra points) Consider a risk-averse agent, with increasing and concave utility $u(\cdot)$ and initial wealth W_0, who faces the risk of having an accident and losing an amount x of her wealth. The agent has access to a perfectly competitive market of risk-neutral insurers who can offer schedules $R(x)$ of repayments net of any insurance premium (which implies the insurer will have a 0 expected payoff). Assume that the distribution of x, which depends on the effort level a to prevent accidents, is following:

$$f(0,a) = 1 - p(a)$$
$$f(x,a) = p(a)g(x)$$

with $\int g(x)dx = 1$ is some probability distribution over x and $p(\cdot)$ is strictly decreasing and convex.

The individual's cost of effort, $c(\cdot)$ is separable from the utility, increasing and convex. So the agent's expected utility is

$$\mathbb{E}_x\Big[u(W_0 - x + R(x)) - c(a)\Big]$$

(a) Suppose there is no insurance market, what action $\hat{a}$ will the agent take?

(b) Suppose that a is contractible, describe the first best contract on payment schedule $R_{\text{FB}}(x)$ and the effort choice a^*.

(c) Suppose a is not contractible, describe the second best payment schedule $R_{\text{SB}}(x)$.

(d) Explain how to implement the second best contract. Would the agent ever have an incentive to misreport an accident? (i.e., report $x = 0$ when $x > 0$)