ECON 200C, Spring 2021

Homework 4

Due: 5/28/2021, before the beginning of the discussion session

Problem 1. In a two consumer, two goods exchange economy, suppose one consumer has a Cobb-Douglas utility and another consumer has utility of $\max(x, y)$.

(a) Are their preferences continuous? Are their demand functions continuous?

Definition for continuity: A preference $\succeq$ is continuous if for all $y \in X$, the sets $\{x : x \succeq y\}$ and $\{x : x \preceq y\}$ are closed sets. Equivalently, that is $\{x : x \succ y\}$ and $\{x : x \prec y\}$ are open sets.

(b) Prove by construction that a Walrasian equilibrium does not **always** exist for this economy. (Give an example that WE does not exist)

(c) What assumption in the proof of existence of Walrasian equilibrium is violated?

(d) Does Walras' Law hold in this economy?

Problem 2. Consider pure exchange economies described with the following preferences and endowments. Draw the Edgeworth box diagrams, including each persons indifference curve through the initial endowment point, Pareto set, and contract curve. Determine all Walrasian equilibrium prices and allocations.

(a)

$$u_A(x_A, y_A) = x_A^{1/3} y_A^{2/3},\ \omega_A = (1, 2)$$
$$u_B(x_B, y_B) = x_B^{1/3} y_B^{2/3},\ \omega_B = (2, 1)$$

(b)

$$u_A(x_A, y_A) = x_A y_A^3,\ \omega_A = (30, 30)$$
$$u_B(x_B, y_B) = 2x_B + y_B,\ \omega_B = (30, 0)$$

(c)

$$u_A(x_A, y_A) = \min\{x_A, y_A\},\ \omega_A = (5, 10)$$
$$u_B(x_B, y_B) = \min\{x_B, y_B\},\ \omega_B = (10, 5)$$

Problem 3. (2018 Final #1) There are only two goods, x and y, and two consumers, A and B in the economy, and no production is possible. The consumers' preferences can be represented by the utility functions $u_A(x, y) = y + \log(1 + x)$ and $u_B(x, y) = y + 2\log(1 + x)$ for all bundles in which $x, y \geq 0$. Each consumer is endowed with 5 units of each good.

(a) Determine all interior Pareto allocations.

(b) Consider all the boundary allocations in which $y_A = 10$ and $y_B = 0$; are any of them Pareto optimal? Why?

(c) Can you make a similar argument for any allocation in which $y_A = 0$ and $y_B = 10$?

(d) Depict all Pareto optimal allocations in an Edgeworth box diagram.

Problem 4. (2018 Final #2) There are r girls and r boys, where r is a positive integer. The only two goods are bread and honey, quantities of which will be denoted by x and y, respectively. Each girl has 8 units of honey but no bread, each boy has 8 units of bread but no honey. Each girl's preference is described by the utility function $u_G(x, y) = \min(\alpha x, y)$, $\alpha \geq 0$ and each boy's preference by the utility function $u_B(x, y) = x + y$.

(a) Determine the aggregate Walrasian excess demand functions for honey and bread.

(b) Determine the Walrasian equilibrium prices and allocations.