ECON 200C, Spring 2021

Homework 5

Due: Before Final Exam

Problem 1. A city government must decide whether to build a library. Rich(R), Bob(B) and Lori(L) are the three citizens.

(a) Assume that the three citizens have following valuations: $\theta_K = 5$; $\theta_B = 15$; $\theta_L = -25$.

(1) Assuming quasi-linear utility forms, What is the efficient decision, to build or not to build the library?

(2) Is an SCF where $d(\hat{\theta})$ is Paretian, $t_i(\hat{\theta}) = 0, \forall\hat{\theta}, \forall i$ implementable?

(3) Describe a Groves-Clarke mechanism for implementing the efficient decision rule.

(b) Repeat step (i)(ii)(iii) with the following valuations: $\theta_K = 5$; $\theta_B = 15$; $\theta_L = -10$.

Problem 2. (First Price Auction with Private Values) Consider a first-price sealed-bid auction of an object between two risk-neutral bidders. Each bidder $i = 1, 2$ simultaneously submits a bid $b_i \geq 0$. The bidder who submits the highest bid receives the object and pays his bid; both bidders win with equal probability in case they submit the same bid. Before the auction takes place, each bidder i privately observes the realization of a random variable t_i that is drawn independently from a uniform distribution over the interval $[0, 1]$. The actual valuation of the object to bidder i is equal to $t_i + 0.5$. Therefore, the payoff of bidder i is given by

$$u_i = \begin{cases} t_i + 0.5 - b_i & \text{if } b_i > b \\ \frac{1}{2}(t_i + 0.5 - b_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_J \end{cases} .$$

(a) Derive the symmetric Bayesian Nash equilibrium for this game (i.e., each bidder uses an equilibrium strategy of the form $b_i = \alpha t_i + \beta$).

(b) What is the conditionally expected payoff of bidder i with type t_i in this equilibrium?

Problem 3. (First Price Auction with Common Values) Consider the first-price auction in Problem 2, except that the actual valuation of the object to bidder i is now equal to $t_i + t_j$ $(j \neq i)$

and therefore the payoff of bidder i now becomes

$$u_i = \begin{cases} t_i + t_j - b_i & \text{if } b_i > b \\ \dfrac{1}{2}(t_i + t_j - b_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_J \end{cases}.$$

Notice that, given his own private type t_i, the expected value of the object is still $t_i + 0.5$, which is the same as in Problem 2.

(a) Derive the symmetric Bayesian Nash equilibrium for this game.

(b) Compare the equilibrium bid rule with Problem 2.

Problem 4. MWG 23.B.2

Problem 5. MWG 23.C.1